# MATH APPS

## MATH APPLICATIONS

Produced by the AP Calculus Class
of
Manhattan High School for Girls

June 2016

# Math Apps

A collection of Math applications
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# Message from the Principal

We are living in unbelievable times with innovations in science, medicine, business, public policy, and the environment exploding all over the place. “Our civilization,” posited Peter Braunfeld of the University of Illinois, “would collapse without mathematics.” Our students need quantitative literacy to understand how things work and how new things could be made to work.

I am so proud of this year’s math journal because it reflects our girls deep understanding of the underpinnings of the math they have studied this year in their AP Calculus course. Thank you to Mrs. Goldie Feinberg, their teacher and our Math Chair, for brilliantly giving our girls the practical applications to conceptual mathematics.

Ms. Estee Friedman
Principal
General Studies

# Foreword

Our students spend many years studying mathematics. In elementary school they focus on arithmetic and number sense, and they move on to tackling deeper mathematical concepts in high school.

What are they left with after all the years of exploring, learning and deciphering?

In *Math Apps*, the AP Calculus class shares some of the applications they found related directly to the high school mathematics they studied the past few years. Each student researched a math topic and discovered how the topic is essential in the fields of science, economics, music, art, computer science or technology. This little book is only a tiny glimpse into how applied math shapes many aspects of our daily lives.

Mrs. Goldie Fienberg
Chair, Math Department

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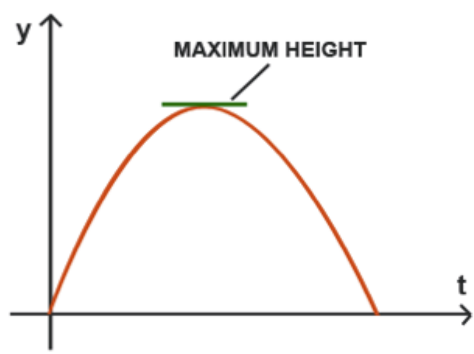
# When Another Water Bottle Doesn't Help

*Aviva Attar*

As we learned in class, the derivative of a function is the slope of the line tangent to the curve, or in simpler English the rate of change of a function at any instant. As seen in the graph, as the function increases, the derivative is positive and and slowly getting smaller, and as the graph reaches a maximum it has a derivative of zero. On the other hand, as the graph begins declining, its derivative is negative and slowly gets larger.

Derivatives are used to help understand how numbers change in many different arenas, specifically economics. For example, total utility of a product is the amount of satisfaction the product gives a person. Marginal utility is the additional utility a person receives from consuming one more unit of that product. Marginal utility describes the rate of change of total utility, hence marginal utility is the derivative of total utility. For example, after drinking three water bottles, the marginal utility of the fourth water bottle is the amount of additional satisfaction gained from the fourth bottle of water. When total utility reaches a maximum point and then begins to descend, the marginal utility, the derivative, is zero. This makes sense in both the context of economics and calculus; when the total utility is zero, a person is no longer gaining any satisfaction from the product, hence the marginal utility is zero. In the example of water, after a person runs a marathon he enjoys the first water bottle very much. Slowly, each water bottle gives him less enjoyment until he is no longer thirsty. At this point, the total utility of water reaches a maximum as the runner



has gained all the satisfaction from drinking water he can possibly achieve. The derivative, or marginal utility, is zero because drinking another bottle of water will not give the runner any more satisfaction. If the runner would continue drinking more bottles of water, he would eventually get sick. Getting sick shows a negative derivative as the enjoyment is negative. Additionally, the more water bottles the runner drinks after getting sick the more rapidly he will get sick. As seen here, math is everywhere! Even though we learned about derivatives in calculus, it can be applied to economics and more!

# A lo(n)g Journey

*Sarah Farber*

Have you ever wondered how ships navigate the seas without getting completely lost? The ocean is a vast, never-ending entity. Winds and currents can easily pull vessels off course, and without the right tools, one might find themselves in America instead of India, like they originally planned.

Many centuries ago, sailors used a process called dead reckoning in order to determine, as best they could, where their ships would end up at the closing of their voyage. They would base their location off the port on the left, attempting to maintain an accurate record of the distance traversed and the direction sailed. Just the slightest miscalculation, and they could end up missing the miniscule island they set sail for by miles.

A few inventions like the sextant and sailing clocks made things easier, but these devices weren't foolproof, and had their own kinks that prevented them from being completely reliable. And so, there was a missing piece in the puzzle of sailing that needed to be filled. Fortunately, a Scottish mathematician was hard at work in his castle inventing logarithms. John Napier spent twenty years coming up with what were known as Napier's logs. The way these logs were set up was not the way they are used to today. [1]

Napier's logs looked like this: $\mathbf{y=log_{nap}(x)}$. In 1614 he produced a book called "A Description of the Wonderful Table of Logarithms" in which he details his work on developing logs. His method was the first of its kind, but still needed work, as doing every computation took more time and effort than most people had. When Napier's book was published, famed Mathematician, Henry Briggs, had read it, and immediately journeyed from London to Edinburgh to tell Napier how to simplify his method. He suggested John switch the form into the one we are so used to today. Briggs and Napier agreed that things

[1] T. (2012, October 11). How does math guide our ships at sea? - George Christoph. Retrieved June 07, 2016, from https://www.youtube.com/watch?v=AGCUm_jWtt4

would be much easier if instead of the "nap" base, a base of ten would be used, and a log of "1" would equal to zero. [2]

Logarithmic equations today look like this: logaN=x. The letter "a" represents the base which is being raised to a certain power, "x." "N" is the outcome of that pairing of "a" and "x". The logarithmic function very simply serves to solve the issue posed when one does not know what power will produce a certain outcome. Because we have calculators today, we do not appreciate the function of logs in dealing with large numbers, as we deal with their exponents instead of the numbers themselves. However, since calculators were not always around, the invention of logarithms was vital to those sailors who couldn't do these calculations with a few clicks of some small buttons.[3]

Sextants and clocks were very expensive due to their hand-crafted nature, so most sailors could not afford these navigational tools and had to figure out their location using intense lunar calculations which would take hours to do. The invention of logs allowed for the appropriate calculations to be done quickly and easily, in order for

[2] Clark, K. M., & Montelle, C. (2011, January). Logarithms: The Early History of a Familiar Function - John Napier Introduces Logarithms. Retrieved June 07, 2016, from http://www.maa.org/press/periodicals/convergence/logarithms-the-early-history-of-a-familiar-function-john-napier-introduces-logarithms

[3] Clark, K. M., & Montelle, C. (2011, January). Logarithms: The Early History of a Familiar Function - John Napier Introduces Logarithms. Retrieved June 07, 2016, from http://www.maa.org/press/periodicals/convergence/logarithms-the-early-history-of-a-familiar-function-john-napier-introduces-logarithms

[4] T. (2012, October 11). How does math guide our ships at sea? - George Christoph. Retrieved June 07, 2016, from https://www.youtube.com/watch?v=AGCUm_jWtt4

sailors to determine their positions at sea. [4]

Thankfully, due to the creation of modern technology, like GPS satellite operated systems, ships are able to easily figure out which tiny speck they are in the gargantuan ocean, and how to get from point A to point B without any mishaps. Even though we do not need to use logarithms at sea anymore, we still appreciate the part they played in many of the most incredible voyages and the role they have taken on in other aspects of our lives. You don't need to be a sailor to appreciate a good logarithm!

# Searching For A Sine

*Draisy Friedman*

If you'd asked me in 11th grade, I never would have said trigonometry had any use beyond answering increasingly far fetched questions on math tests. Yet some quick research reveals a field in which trigonometry's uses are limitless: oceanography, the study of tides. By plotting tides along a sine curve, oceanographers can predict how a tide behaves and when low and high tide will occur.[1]

As I learned in trigonometry, all sine functions have an amplitude and a period. Amplitude refers to the magnitude, or height of the "wave" of a sine function. Period means the distance it takes for the function to complete one full cycle and repeat itself. Different variations of the sine function have different amplitudes and periods.

Because high and low tide tend to occur in periodic patterns, sine curves are perfect for modeling tides at a specific location. For example, let's assume that the height of the water at a New York coast is 4 feet at midnight, and the height of the water at x hours after midnight can be found using $4\sin(2x+\pi)+4$. To find the height of the water at high tide, all one has to do is determine the amplitude of the function and add that to 4. The amplitude of a sine function is the number before sine, so the amplitude of this function is 4. Therefore, at high tide the water along this New York coast is 8 feet high. To find low tide, all you have to do is subtract the amplitude from the original height, meaning the water on the coast is 0 feet high at low tide.

Trigonometry can also be used to figure out how often high tide will occur. All you have to do is look at the period. To find the

[1] http://www.dummies.com/how-to/content/measure-tidal-change-using-a-trigonometry-graph.html, Mary Jane Sterling

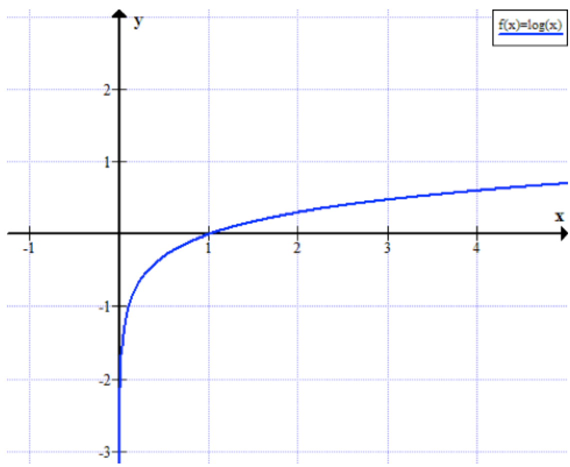
period of a sine function, you divide $2\pi$ by the absolute value of the coefficient of x. Therefore, on this coast high tide occurs every $\pi$ hours.

Of course, this is not a perfect method. Tides change depending on the gravitational pull of the sun and moon, so there is no universal function that can predict tides at every location during every time frame. However, if you only need a general estimate of high and low tide times, sine functions may be the tool you have been searching for.

# Groundbreaking Exponentials

*Leah Genkin*

In an exponential function, some constant b is raised to the power of x to get y. It is written as $y=b^x$. The log function is used to express an exponent in a different form than $y=b^x$. The log function is called the inverse of the exponential function so the x-value of the exponential function becomes the y-value in the log function, and the y-value of the exponential function becomes the x-value in the log function. y=bx is exponential so the log function, its inverse, will be $x=b^y$ which is commonly written as $\log_b x=y$. The common base for the logarithmic scale is a base of 10 ($\log_{10} x=y$). This means that 10 is raised to some y-value to get an x. Just like in an exponential function, in a logarithm function, as x increases, f(x) also increases, but more slowly than in an exponential function. Therefore, the log function is used to measure things that have a great variety of sizes, like earthquakes.[1]



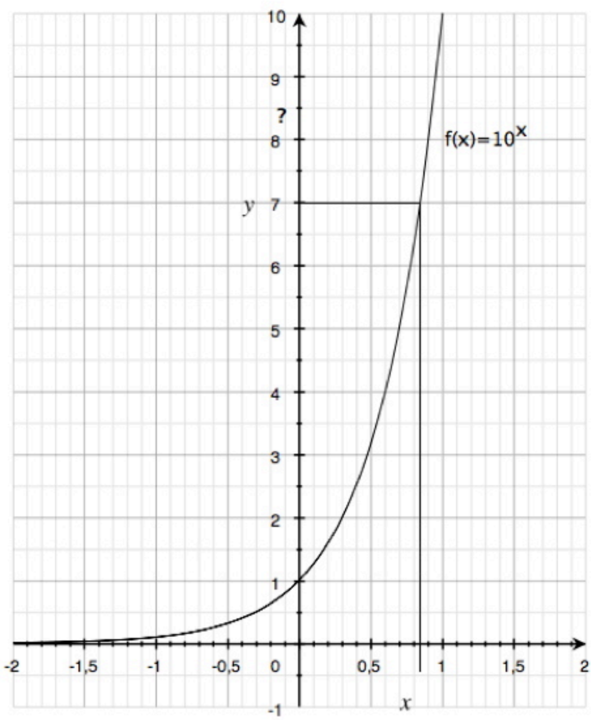

In 1935, Charles Richter used the logarithmic function to develop the Richter scale. When an earthquake occurs, we use this scale to measure its magnitude of how much the ground shook and how much energy was released. The scale runs from 1-10, with one the least in magnitude and ten the greatest.[2]

[1] http://study.com/academy/lesson/using-the-richter-scale-to-measure-earthquakes.html

[2] http://www.montereyinstitute.org/courses/DevelopmentalMath/TEXTGROUP-1-19_RESOURCE/U18_L4_T2_text_container.html

Seismic waves, the vibrations as a result of the earthquake, are measured through an instrument called seismographs. When the ground shakes, the seismograph records the different amplitudes of the vibrations. The logarithm of the amplitude is then taken to determine the magnitude of the earthquake on the Richter scale.[3] So an earthquake with a wave amplitude of 392, has a magnitude of log 392 (which equals 2.6) on the Richter scale.

Since the scale uses the logarithmic function, an increase in one point is really ten times more ground shaking. So an earthquake with magnitude 3, which is just two steps higher than 1, is really 100 times more ground shaking than an earthquake with magnitude 1.




[3] http://earthquake.usgs.gov/learn/topics/measure.php

# Calculus: The Ride of my Life

*Tziporah Hirsch*

When engineers construct a roller coaster model, they use calculus to determine the speed, acceleration, slope, and critical points of the ride. The roller coaster function is like the position graph of an object, depicting the time and position at every point on the graph. The velocity is the derivative of the roller coaster. The velocity curve depicts the speed of the roller coaster "in a given direction at every point."[1] The derivative of the velocity is the acceleration. The acceleration curve depicts "increase in the rate or speed of the ride."[2]

Engineers must find the velocity at the maximum height to make sure the g-force, the pull opposite of gravity, is safe for the riders at that point.[3] The maximum height is when the slope of the tangent line to the roller coaster is zero and the roller coaster changes from increasing to decreasing.

The roller coaster must be continuous and differentiable for the ride to operate.[4] Logically, it must be continuous, without any breaks in the tracks. It must also be differentiable, without any cusps, so the cart of the roller coaster can flow without getting stuck.

In addition, when engineers construct the ups and downs of the roller coaster, they must take into account the acceleration of the ride, for the safety of the riders. Engineers take the derivative of the position to get the velocity. The derivative of the velocity will be the acceleration at that point. Then, they determine whether the acceleration is appropriate for that point in the ride.[5]

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[1] Dictionary.com
[2] Dictionary.com
[3] https://prezi.com/liexwdzq2fzn/calculus-in-roller-coasters/
[4] https://prezi.com/liexwdzq2fzn/calculus-in-roller-coasters/
[5] https://prezi.com/liexwdzq2fzn/calculus-in-roller-coasters/

A roller coaster is usually a piecewise function, constructed with a few different functions. Every function must be continuous and differentiable at the points where the functions meet. Possible functions are parabolas (ax^2+bx+c)[6], for the "drops" of the roller coaster, or various elliptic equations, for the loops of the roller coaster. The loops of a roller coaster cannot be fully circular, because the speed will have to be extremely fast and the g-force will not be able to withstand the speed. The shape of the loop is a clothoid loop,which is shaped like a teardrop. In a clothoid loop, the radius is widest in the middle of the loop, and gets smaller as it gets closer to the ends of the loop. [7]

Roller coaster engineers must determine the slopes of the roller coaster hills to make an accurate model possible for the construction crew to build. The slope will also allow the engineers to determine the speed at specific times throughout the roller coaster. The roller coaster must be at a certain speed to be able to complete a loop. The speed is dependant on the loop. [8]

Calculus is a very important feature in creating a fast, safe, and enjoyable roller coaster.

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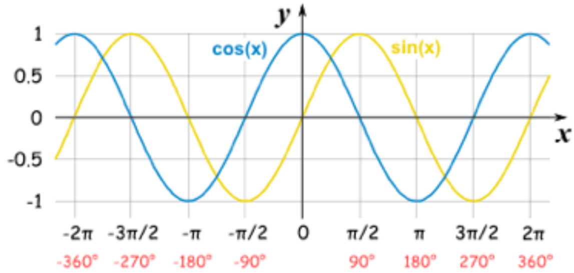
[6] http://www.pleasanton.k12.ca.us/avhsweb/james/calculus/End%20of%20Year/Projects%20and%20other%20Stuff%20New/Roller%20Coaster%20Project.pdf
[7] https://prezi.com/liexwdzq2fzn/calculus-in-roller-coasters/
[8] https://prezi.com/liexwdzq2fzn/calculus-in-roller-coasters/

# Ode to Trigonometry

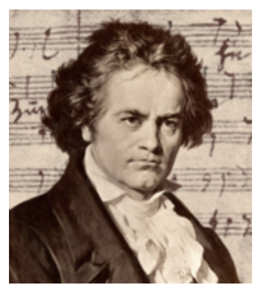
*Ruti Koenig*




We learned about sine and cosine graphs to the right in this year's AP Calculus Class. Transformations of a graph shift and change the picture. The amplitude of a function is half the distance between the minimum and maximum points. The frequency of the graph is the number of full sine curves one can see in 2π radians. Transforming the sine function will change the amplitude and frequency but will preserve the shape of the sine curve.

An application of the sine curve can be found in sound. When one hears a sound, the waves that make up the sounds are comprised of sine and cosine graphs. "Sound waves are very quick changes in air pressure which your ear interprets as sounds. For very pure single tones, a plot of air pressure against time would show them to be sine waves." [1] Different sounds are heard based on the amplitude and the frequency of the wave. The frequency helps interpret the pitch of the sound, while the amplitude dictates the loudness.

Beethoven was one of the most prominent musicians who ever existed, despite the fact that he was deaf for most of his career. How was he able to make such beautiful music if he could not hear? The sound patterns of his music were the key to his success. For example, his famous Moonlight Sonata begins with repetitive notes in sets of three. Although repetitive patterns seem uncomplicated, every triplet is made up of



[1] http://www.mathopenref.com/trigsinewaves.html

complex calculations using the sine curve's frequency and amplitude. Beethoven once said "I always have a picture in my mind when composing and follow its lines." [2]

The pitch frequencies of different notes form a geometric series. The sine wave of each note can be determined, allowing a person to see the musical patterns that Beethoven could not hear. Beethoven understood that when certain notes are played together they sound amazing, however, other notes sound awful. When the difference of the frequencies have a constant number the music sounds pleasing to the ear, which is called consonance. On the other hand, when the difference of the frequencies is not a constant number, the notes sound rough, full of tension and unpleasant to the ear, which is called dissonance. Beethoven used both of these techniques in his music depending on the mood of his piece.

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[2] St. Clair, Natalya. (2014, September). Music and math: The genius of Beethoven [Video file]. Retrieved from http://ed.ted.com/lessons/music-and-math-the-genius-of-beethoven-natalya-st-clair.

# Computers 101

*Rivky Kreiser*

In today's world we can all admit that computers have largely impacted our lives, but do most of us really know anything about how they work? We hear things like ones and zeros or ons and offs but what do they really mean?

The binary number system is a base two number system that only makes use of two digits: 0 and 1. This is important because machines cannot count numbers on their fingers (like many humans tend to do) but they can understand the difference between when a switch is open or closed or when an electrical flow is on or off.

| Binary | Decimal |
|---|---|
| 0 | 0 |
| 1 | 1 |
| 10 | 2 |
| 11 | 3 |
| 100 | 4 |
| 101 | 5 |
| 110 | 6 |
| 111 | 7 |
| 1000 | 8 |
| 1001 | 9 |
| 1010 | 10 |
| 1011 | 11 |

Using a base two system is not new. Aboriginal people in Australia counted by two and many African tribes "sent complex messages using drum signals at high and low pitches (1)." In more modern times, binary was used again in Morse code which uses dots and dashes to represent the alphabet.

In our minds, we count using a base ten number system. However, it is not hard to convert into a base two number system. In first grade we learned about the ones column, the tens column and so on. In math class, we learned how to reorient the values of our number columns. For binary, we replaced the tens column with a twos column and the hundreds column with a fours column (See chart).

Today, digital computers operate based on two possible states: on and off. On is represented by the number one and literally means that an electrical current is present. Off is represented by the number zero

[1] Redshaw Kerry. "Binary - So Simple a Computer Can Do It."kerryr.net. n.p. n.d. Web. 30 May 2016.

and literally means that an electrical current is absent. "Each binary digit, or bit, is a single 0 or 1, which directly corresponds to a single 'switch' in a circuit. Add enough of these 'switches' together, and you can represent more numbers (2)."

But why can't computers use a base ten number system? The answer is simple. The more possibilities in existence, the more complicated something becomes. Furthermore, modern switches are not capable of holding ten possibilities, but they are very capable of holding two possibilities. So here we are today!

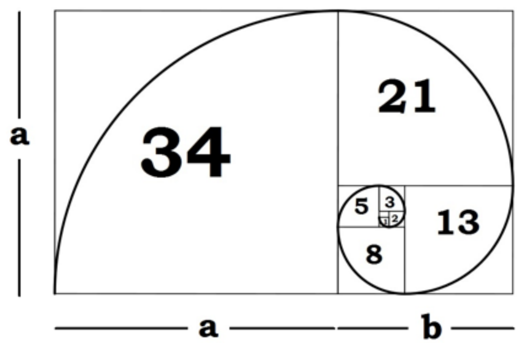
[2] Blinnikov Ilyosha. "Why Computers Use Binary." Nookkin.com. n.p. 15 May 2010. Web. 30 May 2016.

# Beautiful Math

*Shira Nabatian*

The golden ratio is a number found by dividing a line segment into two parts such that the longer segment divided by the smaller segment is equal to the whole line segment divided by the longer segment. This is something in the golden ratio. So a divided by b is equal to a plus b, divided by a, which equals 1.618033988749894820 …[1] It is symbolized by phi, the Greek letter.

In class we learned about sequences, or a list of numbers in a special order.[2] One of the sequences we learned about was the Fibonacci sequence. The Fibonacci sequence is: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, and so on. The next number of the Fibonacci sequence is found by adding the two numbers that come before it. For example 0 plus 1 equals 1, and 1 plus 1 is two.

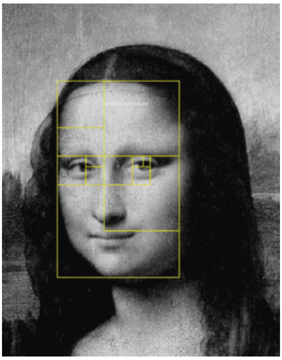



The ratio of any two consecutive numbers in the Fibonacci sequence are slowly approaching the golden ratio. 3/2 is 1.5 and 13/8 is 1.625, and so on, slowly getting closer and closer to the golden ratio, 1.618033988749894820...

A contemporary of Da Vinci, Luca Pacioli said, "Without mathematics there is no art."[3] Tools, such the golden ratio can help create beautiful works of art and architecture. The golden ratio creates images and structures pleasing to the eye and therefore has been used for centuries.

For example, The Mona Lisa is beautiful, and her face was created

[1] http://www.livescience.com/37704-phi-golden-ratio.html
[2] https://www.mathsisfun.com/definitions/sequence.html
[3] http://www.goldennumber.net/art-composition-design/

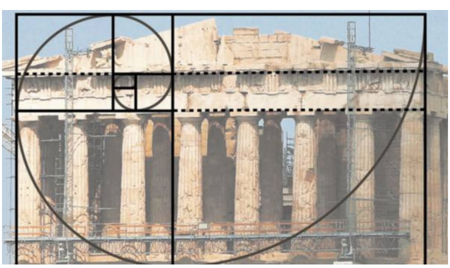


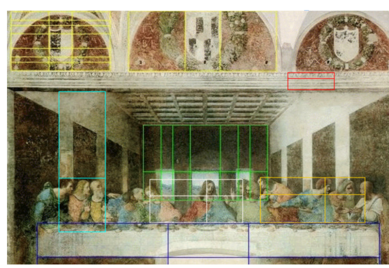
by Da Vinci, around the golden ratio.

Da Vinci also created the "Last Supper" based on the golden ratio.

Architecture has also been created using the golden ratio. The Parthenon, a temple in ancient Greece, was built with dimensions of the golden ratio.

The Fibonacci sequence and the golden ratio are more than just math ideas. They are a tool used in art to create beautiful masterpieces. Orthodontist Mark Lowey, at The University College Hospital in London did a study taking detailed measurements of models' faces. He was proving how people find people prettier whose faces are closer in proportion to the golden ratio. Art and buildings are still built based on Fibonacci and the golden ratio. [4]





[4] https://plus.maths.org/content/golden-ratio-and-aesthetics

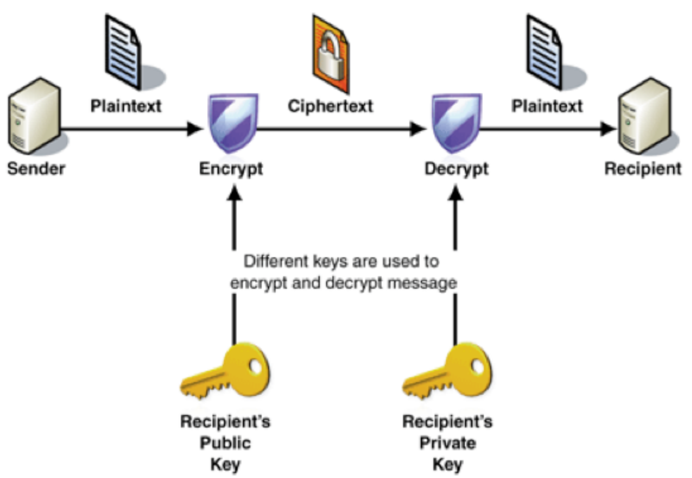
# Public Key Infrastructure: Securing the Virtual World

*Tikva Nabatian*

Public Key Infrastructure is a two key encryption system that is used to provide authentication and confidentiality in data transmissions such as online transactions. The system is made up of two keys that are mathematically related.

The two keys used in this encryption system are the public key and the private key. The public key is available to everyone and is used to generate the encrypted message. The private key is only available to the person receiving the encrypted message and is used to decrypt the message.

PKI works as follows: two large prime numbers, say P1 and P2, are multiplied to produce some C. The C is then used as the public key, which is available to everyone. This is the key that is used to encrypt the transmission. P1 and P2 are the private key and must be kept secret because it is used to decrypt the transmission.[1]



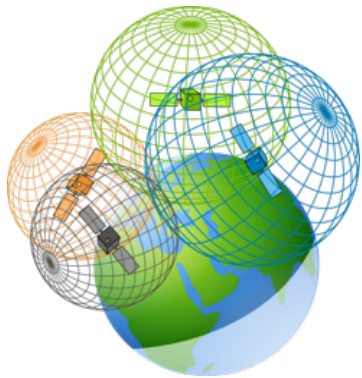

[1] Albarqi, A., Alzaid, E., Al Ghamdi, F., Asiri, S. and Kar, J. (2015) *Public Key Infrastructure:* A *Survey*. Journal of Information Security ,6, 31-37.

The security of the system rests on the difficulty of prime factorization. Meaning it is difficult to discover the private key from the public key because given some C there is no simple and fast method for factoring out P1 and P2. This makes PKI a reliable system for confidential transactions, such as buying clothing online and sending classified documents. [2]

[2] Crow, Jerry. *Prime Numbers in Public-Key Cryptography*. Rep. SANS Institute, 2003. Print.

# Navigating Intersections: The Math of GPS

*Shoshana Rosenthal*



We're pretty much dependent on GPS (Global Positioning Systems) to get basically anywhere. The entire GPS system relies on geometry and algebra to calculate position, specifically through systems of equations and a process called "trilateration."

There are 32 GPS satellites[1] orbiting 12,551.7 miles above the Earth.[2] Those satellites send down radio signals containing the satellite's position and the time at which the signal was sent.[3] The radio signals travel at the speed of light (186,282 miles per second)[4], though the Earth's atmosphere does slow them down. The receiver compensates for this by taking the interference from the average atmospheric density and thickness into account when calculating the amount of time between when the satellite sent the signal and when the receiver picked up the signal.[5] Multiplying the time at which the signal was sent by the speed of the signal gives the distance between the satellite and the receiver picking up the signals.[6] That distance forms the radius of an imaginary sphere.

The intersection of two spheres is generally a circle (unless it is a single point). A third intersection sphere will intersect that circle at one or two points.[7] The receiver uses three signals to form three intersecting imaginary spheres, narrowing down its position to two possibilities, and a fourth signal forming a fourth intersecting imaginary sphere is

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[1] http://tycho.usno.navy.mil/gpscurr.html
[2] http://www.faa.gov/about/office_org/headquarters_offices/ato/service_units/techops/navservices/gnss/faq/gps/
[3] https://www.math.hmc.edu/funfacts/ffiles/10002.2-8.shtml
[4] http://www.gps.gov/multimedia/tutorials/trilateration/
[5] http://www.maa.org/sites/default/files/pdf/cms_upload/Thompson07734.pdf
[6] http://www.maa.org/sites/default/files/pdf/cms_upload/Thompson07734.pdf
[7] https://www.math.hmc.edu/funfacts/ffiles/10002.2-8.shtml

usually enough to pinpoint the exact position of the receiver: the point at which all four imaginary spheres intersect.[8]

This system requires the clocks on all of the satellites and on the receiver to be perfectly synchronized. Since the clock on the receiver is often not aligned with those on the satellites, the receivers account for the error using algebra. The receiver's clocks error is represented by $\varepsilon$. Positive $\varepsilon$ means that the receiver's clock is ahead of the satellites' clocks and negative $\varepsilon$ means that the receiver's clock is behind the satellites' clocks. As long as the same receiver is used, $\varepsilon$ will be a constant variable, which can be found through a system of equations.

Systems of equations can be solved through substitution, addition of the equations, or matrices. Computers can solve these systems of equations very quickly. Once $\varepsilon$ is found, it is plugged in and a second system of equations is used to calculate the coordinates of the receiver's location.[9]

Before the last century, accurate navigation involved applying a lot of trigonometry, but the advent of GPS receivers doing the math for us allows for anyone to navigate with relative ease, even without a map.

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[8] https://www.math.hmc.edu/funfacts/ffiles/10002.2-8.shtml

[9] http://www.maa.org/sites/default/files/pdf/cms_upload/Thompson07734.pdf

# "Cool" Calculus

*Chaya Sherman*

Newton's Law of Cooling states that the rate of change of the temperature of an object is proportional to the difference between its own temperature and the ambient temperature (i.e. the temperature of its surroundings).[1] As we proved in AP Calculus when we were learning differential equations, this law is a non-standard decay function. It is often used by forensic scientists to estimate the time of death based on body temperature upon discovery and the temperature of the room.

It was discovered when Newton observed the relationship between the temperature of an object and how the temperature decreases based on the temperature of the room around it. He solved the differential

$$\frac{dT}{dt} = -k(T - T_a)$$

to find the function

$$T(t) = T_a + (T_0 - T_a)e^{-kt}$$

For instance, a hot cup of coffee in a freezer cools fastest when it is first put in and the difference between the temperature of the coffee and the freezer is greatest. As the coffee cools and the difference


[1] http://www.ugrad.math.ubc.ca/coursedoc/math100/notes/diffeqs/cool.html

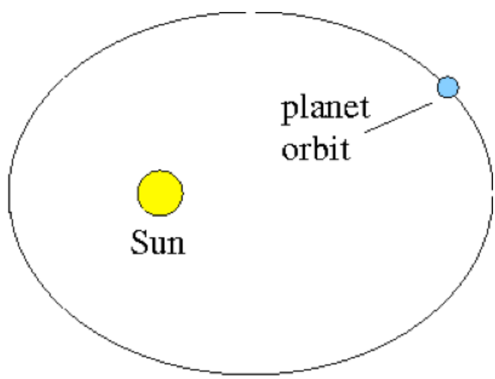
between the coffee's temperature and the freezer's temperature lessens, the temperature of the coffee decreases more and more slowly.

Newton's law can also be used to find the temperature of the body at any time following death, when the body's temperature first began to drop. The resulting time is approximate, because scientists assume the standard body temperature was 98.6 degrees Fahrenheit, and variations in the body's initial temperature can result in a slightly flawed time of death.

# As the World Turns

*Miriam Wilamowsky*

Using a mathematical definition, an ellipse is "a closed plane curve generated by a point moving in such a way that the sums of its distances from two fixed points is a constant."[1] In class, we learnt about the equations of ellipses and how to graph them while taking into account their distance from the foci. Ellipses have a major application beyond the math classroom. The entire galaxy works through a system of elliptical motions. The motions of the planets are elliptical as shown in the image below, where the Sun is the focus point of the orbit.




The Earth, and every other planet, has two constant elliptical rotations. The Earth is constantly rotating upon its axis, but it is also moving around the Sun in the same direction as its own rotation. It takes roughly one day for the Earth to rotate about its axis, while it takes a full year for the Earth to rotate around the Sun. Because the Earth is rotating upon its axis, in order for every spot on Earth to rotate about its axis and yet still come back to the same spot with proportion to the Sun, it must turn a little extra because of the Earth's motion around the Sun. Therefore, the Earth turns a little more than once, with respect to the stars (which do not rotate about the Sun), in order to complete a full rotation with respect to the Sun. The "little bit more" is the exact angle that the Earth rotates around the Sun in one day.[2] This angle averages a little less than one degree and is illustrated in the figure on the next page.

It takes Earth about four minutes to turn this angle. However, the difference is not always the same because the Earth does not move

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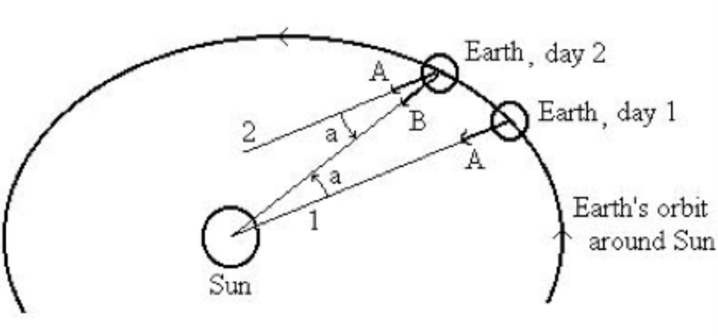
[1] Merriam-Webster; www.merriam-webster.com

[2] Simon Radford; http://www.cso.caltech.edu/outreach/log/NIGHT_DAY/elliptical.htm




in a circular path around the Sun, rather an elliptical path. It happens to be that the Earth is closer to the Sun in January than in July. The average difference of the Earth to the Sun in 93 million miles and the difference of the distance in January and July is three million miles. The speed that the Earth rotates on its axis is dependent on the Earth's proximity to the Sun. The closer the Earth is to the Sun the faster its speed about its axis. Since the Earth is closest to the Sun in January and farthest in July, the Earth is moving faster about its axis in January than in July. Therefore, in order for the Earth to return to the same exact spot with respect to the Sun each year, the Earth must rotate a little more each day in the winter months to return to the same spot with respect to each point's direction to the Sun. This occurs because in the winter months the Earth is farther from the Sun and therefore revolves around its axis slower so it must rotate a little more each day to make up for its slower speed. The opposite applies for the summer months. The small amount per day that the Earth must rotate in the winter months accumulates to 7.7 minutes. However, the Earth rotates a little less each day during the summer months and this decrease accumulates to 7.7 minutes, as well. So, the increased rotation and decreased rotation cancel each other out after a full year.[3] Overall, the elliptical motion of the orbit affects how the earth rotates and the different movements it must make at different times of the year in order to return to the same spot with respect to the Sun each year.

[3] Simon Radford; http://www.cso.caltech.edu/outreach/log/NIGHT_DAY/elliptical.htm

*"For the things of this world cannot be made known without a knowledge of mathematics."*

*- Roger Bacon*